ECC 2012, 10/30/2012

QUERETARO, MEXICO

Non-uniformity

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Based on joint work with Alfred Menezes – see http://anotherlook.ca especially "Another look at HMAC" and "Another look at Non-uniformity."

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2. How this controversy ties in with ECF & ECDLP.

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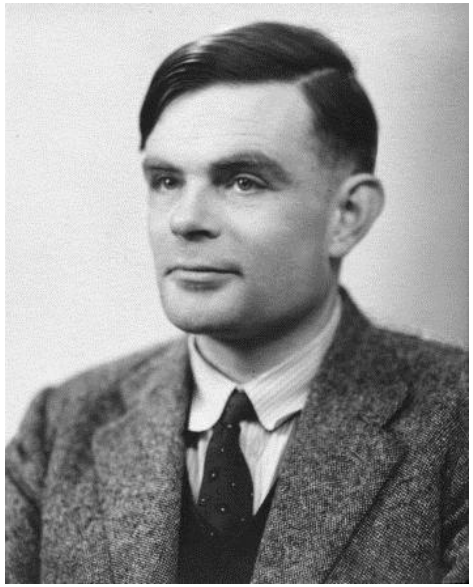

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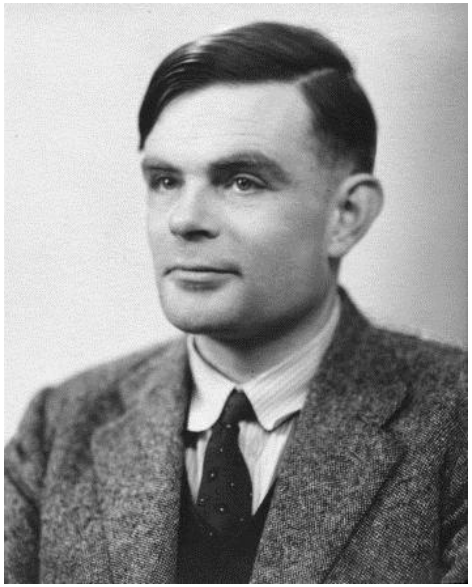

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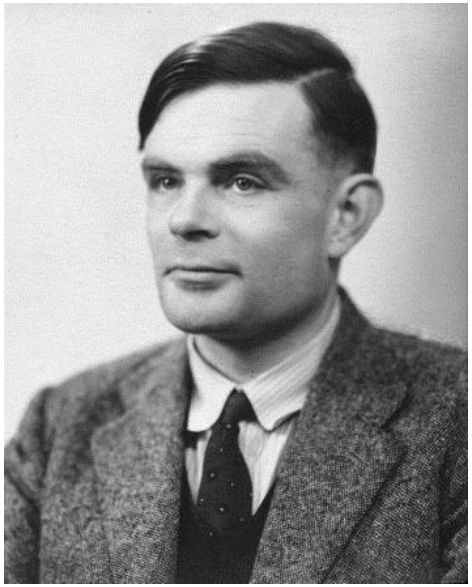

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Our work on HMAC and non-uniformity can be viewed as a vindication of Turing's viewpoint in the setting of provable security in cryptography.

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It is used to construct the hash-based message authentication code HMAC.

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After Wang (2005) showed how to find collisions for MD5 (and similar results were likely for SHA1), the 1996 proof lost its usefulness as a real-world security guarantee.

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The following summary is informal and much simplified, but captures essentially what is going on.

At a key point in the proof Bellare describes an adversary of the prf-property of the compression function f that arises when HMAC fails to have the desired security property.

He sets (M*,M´*) equal to a pair of b-bit messages for which the probability of a collision f(K,M*) = f(K,M´*) (where the probability is taken over all keys K in $\{0,1\}^c$) is maximal.

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This algorithm A exists in the mathematical sense, because the pair (M*,M´*) obviously exists. But it is unconstructible in the sense that there is no feasible way to construct it.

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(2) Algorithms are a computational notion. What does it mean in practice for an algorithm to exist mathematically if it's unconstructible?

(3) How can one conclude anything at all about the real-world security of HMAC if one has to assume that the compression function resists even unconstructible adversaries?

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Bellare is also coinventor (along with Rogaway) of the paradigm "practice-oriented provable security," and the introduction to his paper stresses the real-world applicability of results on the security of HMAC, which is deployed far and wide. It is surprising that someone like that would prove a theorem by means of what we believed to be a bogus argument.

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Within hours of the paper's posting on eprint, we received an angry email from Bellare objecting to our use of the word "flaw" and saying that his proof is correct in the non-uniform model of complexity.

In response to my question about why his paper made no mention of the fact that the main theorem was intended to be understood in the non-uniform model and is invalid in the uniform model of complexity, Bellare wrote:

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A few other leading theoretical cryptographers were also angry at us, and supported Bellare.

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From Jonathan Katz's blog (February 28, 2012): "Many researchers are justifiably concerned about the fact that Alfred Menezes will be giving an invited talk at Eurocrypt 2012…. I share this concern."

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At this point we reexamined other parts of Bellare's paper – especially his method for concluding that his main theorem justifies HMAC up to $2^{c/2}/n$ queries – with non-uniformity in our minds.

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Thus, if you plug in the best attacks known on the pseudorandomness of the compression functions used in real-world deployments of HMAC – and it's reasonable to assume that these compression functions were designed well enough so that only generic, general-purpose attacks are available – you can solve for the bounds on the parameters for which the theorem loses content.

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But recall that Bellare insisted that he is working in the non-uniform model of complexity, and even claimed that any concrete study of security should be in that model (and that it had never occurred to him that any reader would fail to understand this).

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To explain this, I will enlist the help of the football monkey.

The football monkey is a cousin of the proverbial monkey-at-a-typewriter, who, given enough time, can randomly type a flawless *Macbeth,* but who in practice is unlikely to get to the end of "Enter three witches…"

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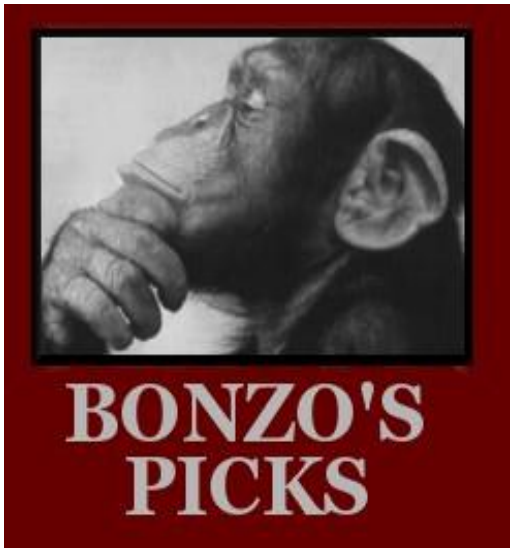
Much like human football pundits, the football monkey is very proud and pleased with himself whenever he does better than 50% / 50%.

The football monkey's non-uniform attack on NFL pseudorandomness:



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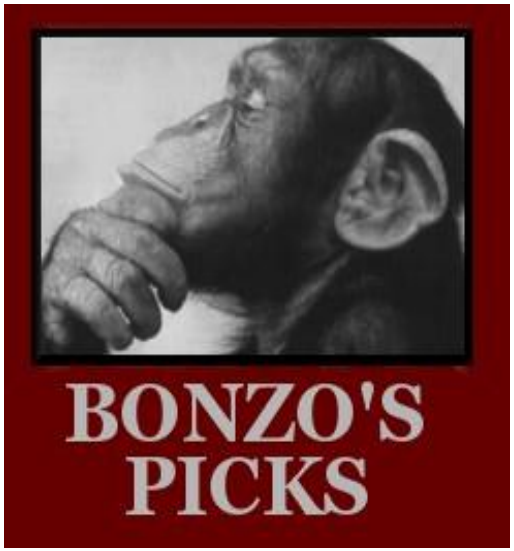

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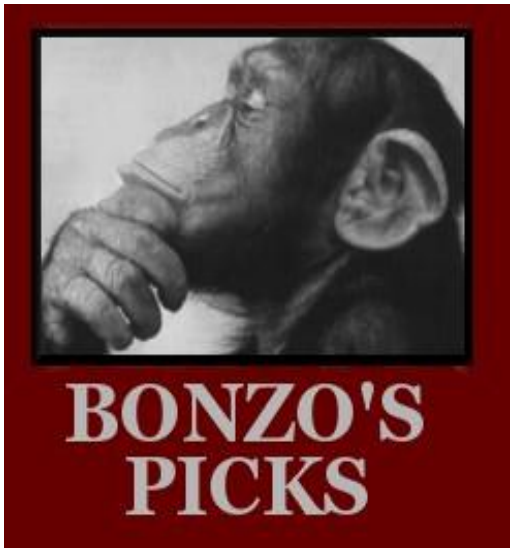

Suppose the season has 256 games (not quite true). Under this attack the monkey will get at least 144/112 right/wrong rather than 128/128.

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This could mean 112/144 rather than 144/112, but we get to pick the monkey's best season.

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In reality, the football-monkey attack, far from being trivial and stupid, is how Alfred and I showed that the main theorem of Bellare's Crypto 2006 paper on HMAC is completely worthless in practice.

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Namely, the football monkey can be used to show the existence of a generic prf-adversary whose advantage is much, much greater than exhaustive key search.

For any subset S of {1,2,…,c}, any message M in $\{0,1\}^b$ and any bit t, define the bit-valued function $u_{S,M,t}(K)$ for variable K in $\{0,1\}^c$ to be

1 if the XOR sum of the S-indexed bits of f(K,M) is equal to t; 0, otherwise.

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Here's our adversary A: It has (S*,M*,t*) wired into it. It makes 1 query, namely M*, to O, which answers O(M*). If the XOR sum of the S*-indexed bits of O(M*) is equal to t*, then A guesses that O is really f(K,.); otherwise A guesses that O is random.

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To resist the adversary A a compression function f(K,M) would need to have the XOR sum of any subset of its bits much more evenly split between 0's and 1's (for arbitrary fixed M and varying K) than would be expected of a random function. No one would think that the compression functions used in MD5, SH1, AES256, etc. have such a property. Thus, we can regard A as a generic adversary.

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Instead of justifying HMAC security up to 2^{44} queries for MD5 and up to 2^{60} queries for SHA1, because of the football-monkey adversary Bellare's theorem loses content for MD5 after 2^{20} queries and loses content for SHA1 after 2^{28} queries!

From Bellare's 24 February 2012 email to me:

"…if you want your series [of Another Look papers] to gain respect among theoretical cryptographers, it would benefit from reflecting our feedback and being better informed about the basics of the field.

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But we weren't the ones who failed to understand the dramatic consequences of eschewing Turing machines and opting instead for "Turing machines with advice."

Clint Eastwood

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The Clint Eastwood character is called

Dr. Frank Corvin,

and Ethan is one of the young guys.

James Garner, Tommy Lee Jones, Donald Sutherland, Clint Eastwood

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Frank: Maybe you ought to get your money back.

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Note that "input" includes the specification of all parameters needed to describe the problem instance, even if some of them are fixed or vary in a small range in the application we have in mind.

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In his textbook on the foundations of cryptography, Goldreich emphasizes this. He points out that typically a problem has exponentially many inputs of a given bitlength, and a single advice-string is not likely to be helpful for solving exponentially many problem instances.

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Example (the Cunningham Project): Factor $b^n \pm 1$ for b=2,3,5,6,7,10,11,12 and large n.

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1. Oracle complexity of factoring

Ueli Maurer (1995) considered "the problem of factoring integers in polynomial time with the help of an infinitely powerful oracle who answers arbitrary questions with yes or no… The goal is to minimize the number of oracle questions."

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General idea: Randomly choose a mod-N elliptic curve and ask the oracle how to change the bits in the coefficients of the curve's equation so as to get a curve that has smooth order mod p (where p is a prime factor of N). After that, Elliptic Curve Factorization will take only polynomial time.

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This is a nice result even though it has no use for practical cryptography.

2. Maurer-Wolf's DHP-DLP equivalence

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Again using elliptic curves of smooth order, they show that "for every cyclic group G there exists a small piece of information, which depends only on the order of G, that makes breaking the Diffie-Hellman protocol and computing discrete logarithms equivalent in G."

This is a wonderful result, but they incorrectly term it "non-uniform equivalence". The advice-string depends on the order of G, not just on the bitlength of the order of G.

As the above examples show, it might be very interesting to consider algorithms with advice-strings even if the algorithms are not non-uniform.

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Even worse, every uniform algorithm is also a non-uniform algorithm (with empty advice-strings).

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(2) Give a polynomial-time algorithm that, given any elliptic curve E defined over a finite field and a polynomial-size advice-string that depends only on E, computes discrete logs on E.

(3) The same as (2) with either or both occurrences of "polynomial" replaced by "subexponential", or by $N^{1/4}$, where N is the order of the group E.

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But suppose we are interested only in a restricted problem, such as $ECDLP_{ABC}$, the problem of finding discrete logs on anomalous binary curves. There is only one ABC curve for each even-degree extension of F_2 and two for each odd-degree extension of F_2. Then solutions to (2) or (3) will give non-uniform algorithms.

Finally, I want to mention a valiant but unsuccessful attempt to solve an open problem similar to (1) above but for the decision Diffie-Hellman problem rather than the discrete log problem.

3. The torsion-subgroup “non-uniform” attack on Decision D-H

Q. Cheng and S. Uchiyama, *Nonuniform polynomial time algorithm to solve decisional Diffie-Hellman in finite fields under conjecture,* CT-RSA 2002.

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They attempted to solve the open question:
Is it possible, given a polynomial-size hint depending on the primes p and n (where n|(p-1)) but not on the rest of the input, to solve any instance of DDH in the order-n subgroup of $(F_p)^*$?

Note that even if they could give an affirmative answer to that question – which they claimed to have done under a plausible conjecture – it would not have given a non-uniform ptime DDH algorithm.

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Alfred and I analyzed their algorithm in *Math. Comp.* 73 (2004), pp. 2027-2041.

The advice-string they needed was an elliptic curve defined over a number field K that has a torsion-point of order n. The only evidence for their conjecture that such curves exist – with psize [K:Q], psize generating polynomial for K, psize coefficients of the curve, etc. – is the fact that mathematicians have not yet been able to prove that they do not exist.

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However, there are heuristic arguments and indirect evidence (partial results) that make it seem extremely doubtful that such curves exist.

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"Of course, in the absence of a proof that such a point [modular point corresponding to the required high-torsion curve] cannot exist, one is free to conjecture whatever one's heart desires, just as one is free to conjecture the existence of little green men walking across the ice fields of Ganymede."

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Ganymede

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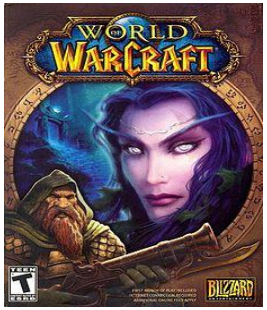

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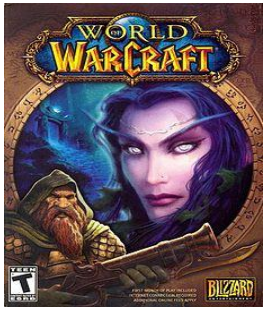

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Following G. H. Hardy, we should be happy to be working on something "gentle and clean".

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There is nothing shameful about working on problems that have great aesthetic appeal and zero practical utility.

Following G. H. Hardy, we should be happy to be working on something "gentle and clean".

After all, Fermat's Last Theorem has absolutely no practical utility whatsoever.

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provided that no claim is made about its practicality.